Lecture 11: Introduction to QCD

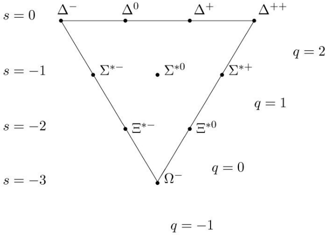
Sept 29, 2016

Outline

- Why Color?
- From Color to the QCD LaGrangian
- The Running of α_s
- Implications for $e^+e^- \rightarrow$Hadrons
- Discovery of Jets
- Describing quark hadronization

Why Color (I)

Imposition of Fermi Statistics

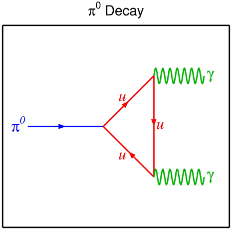


$\Delta^{++} = uuu$: Identical particles

- spin$=3/2$: Symmetric under interchange
- s-wave ($\ell = 0$): Symmetric under interchange
- Need another degree of freedom to antisymmetrize

Need at least 3 possible states to antisymmetrize 3 objects

$\pi^0 \to \gamma\gamma$

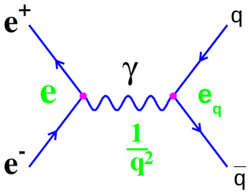


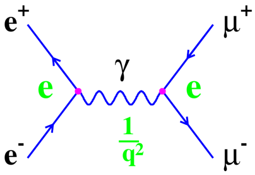
Decay process through an internal quark loop

$$\Gamma \propto N_C^2 (Q_U^2 - Q_d^2)^2$$

Consistent with 3 colors

Why Color (II): $e^+e^- \rightarrow hadrons$



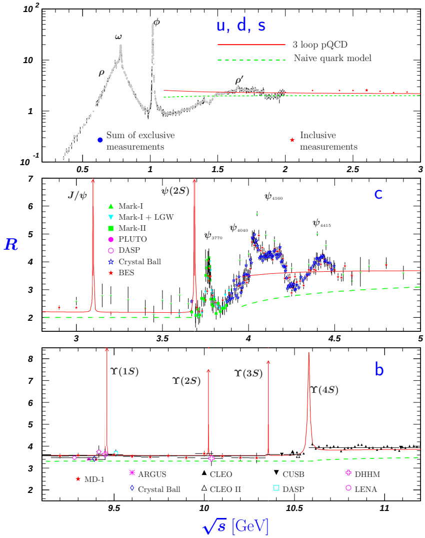


- Describe process $e^+e^- \rightarrow hadrons$ as $e^+e^- \rightarrow q\overline{q}$ where q and $\overline{q}$ turn into hadrons with probability=1
- Same Feynman diagram as $e^+e^- \rightarrow \mu^+\mu^-$ except for charge. To lowest order (no QCD corrections)

$$R = \frac{\sigma(e^+e^- \rightarrow hadrons)}{e^+e^- \rightarrow \mu^+\mu^-} = N_C \sum_q e_q^2$$

where N_C counts number of color degrees of freedom and sum is over all quark species kinematically allowed

$e^+e^- \to hadrons$: Measurement of R



$$R \equiv \frac{\sigma(e^+e^- \to hadrons)}{e^+e^- \to \mu^+\mu^-}$$
$$= N_C \sum_q e_q^2$$

where N_C is number of colors

- Below $\sqrt{s} \sim 3.1$ GeV, $R = 2$

  Only u, d, s quark-antiquark pairs can be created

$$\sum_q e_q^2 = \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2$$
$$= \frac{6}{9} = \frac{2}{3} \Rightarrow N_c = 3$$

- Above 3.1 GeV, charm pairs produced; R increases by $3(\frac{2}{3})^2 = \frac{4}{3}$
- Above 9.4 GeV, bottom pairs produced, R increases by $3(\frac{1}{3})^2 = \frac{1}{3}$

From Color To QCD (I)

- R tell us $\exists$ 3 colors, but doesn't tell us anything about the color force.
- Theory of Strong Interactions QCD developed in analogy with QED:
  - Assume color is a continuous rather than a discrete symmetry
  - Postulate local gauge invariance
  - Describe our fundamental fermion fields as a 3-vector in color space

$$\psi = \begin{pmatrix} \psi_r \\ \psi_b \\ \psi_g \end{pmatrix}$$

  - Let's take SU(3) as our the candiate for the rotation group for this 3-space

$$\psi'(x) = e^{i\lambda^i \alpha_i/2}$$

where the λ^i are the 8 SU(3) matrices we already know

From Color To QCD (II)

- Impose local Gauge Invariance by introducing terms in A_μ and the quark kinetic energy term ∂_μ:

$$\begin{aligned} A\mu &\rightarrow A\mu + \partial_\mu \alpha \\ \mathcal{D}_\mu &\equiv \partial_\mu - i\frac{g}{2}\lambda_a A^a_\mu \end{aligned}$$

  where A_μ is a 3×3 matrix in color space formed from the 8 color fields and λ_i are the SU(3) matrices and i goes from 1 to 8

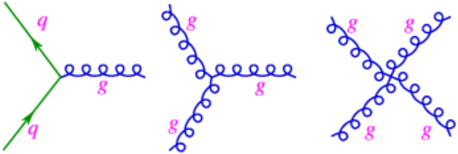
- The tensor field is:

$$\begin{aligned} G_{\mu\nu} &= \frac{1}{ig}[\mathcal{D}_\nu, \mathcal{D}_\mu] = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\nu, A_\mu] \\ G^a_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + f_{abc} A^b_\mu A^c_\nu \end{aligned}$$

  This plays the same role as $F_{\mu\nu}$ in QCD

- Note: unlike QED, there are several A fields and these A don't commute!
  - This means that the gluons have color charge and interact with each other
  - Note that there is no color singlet gluon

The QCD Feynman Diagrams



- qqg vertex looks just like $qq\gamma$ with $e \to g$
- Three and four gluon vertices
  - Three gluon coupling strength gf^{abc}
  - Four gluon coupling strength $g^2 f^{xac} f^{xbd}$

where

$$\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right] = if_{abc}\frac{\lambda_c}{2}$$

and $f_{123} = 1$, $f_{147} = f_{246} = f_{257} = f_{345} = \frac{1}{2}$, $f_{156} = f_{367} = -\frac{1}{2}$, $f_{458} = f_{678} = \sqrt{3}/2$.

The Running of α_s (I)

- In calculating R, assumed that strong interactions didn't significantly affect the cross section; derived this using impulse approximation
  - Quarks act as if they are free during the EM interaction
- Seems odd since α_s is large, as measured via the decay widths of strong decays
- Great success of QCD is ability to explain why strong interactions are strong at low q^2 but quarks act like free particles at high q^2
- Coupling constant α_s *runs*; It is a function of q^2

Low q^2	α_s large	"confinement"
High q^2	α_s small	"asympotic freedom"

- This running is not unique to QCD; Same phenomenon in QED
  - But α runs more slowly and in opposite direction
  - Eg at $q^2 = M_z^2$, $\alpha(M_Z^2) \sim 1/129$
- Running of the coupling constant is a consequence of *renormalization*
- Incorporation of infinities of the theory into the definitions of physical observables such as charge, mass

The Running of α_s (II)

- QED and QCD relate the value of the coupling constant at one q^2 to that at another through renormalization procedure

$$
\begin{aligned}
\alpha(Q^2) &= \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log\left(\frac{Q^2}{\mu^2}\right)} \\
\alpha_s(Q^2) &= \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi} (33 - 2n_f) \log\left(\frac{Q^2}{\mu^2}\right)}
\end{aligned}
$$

- In the case of QED, the natural place to measure α is clear: $Q^2 \to 0$
- Since α_s is large at low Q^2, no obvious μ^2 to choose
- It is customary (although a bit bizarre) to define things in terms of the point where α_s becomes large

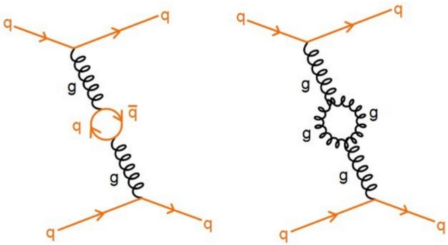
$$
\Lambda^2 \equiv \mu^2 \exp\left[\frac{-12\pi}{(33 - 2n_f)\,\alpha_s(\mu^2)}\right]
$$

- With this definition

$$
\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f)\log(Q^2/\Lambda^2)}
$$

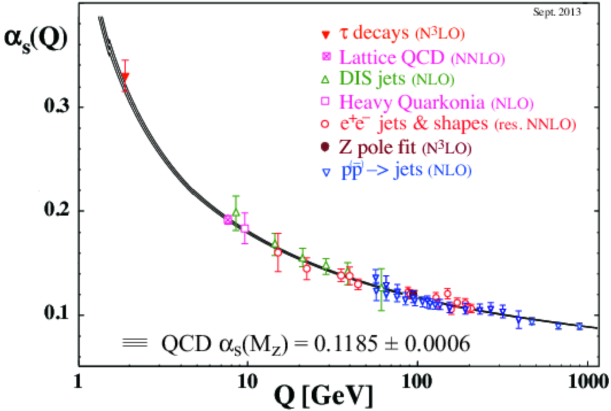
  - For $Q^2 \gg \Lambda^2$, coupling is small and perturbation theory works
  - For $Q^2 \sim \Lambda^2$, physics is non-perturbative
- Experimentally, $\Lambda \sim$ few hundred MeV

Why do coupling constants run?



- Higher order loop corrections in propagator
  - Photon propagator only has fermion loops
  - Gluon propagator also has gluon loops
  - Fermion and gluon loop terms opposite have opposite sign
  - Hence running depends on number of flavors
- Must perform renormalization to remove unphysical infinities

Measurements of α_s



We'll talk more about how these measurements over next few weeks

Implications of the Running of α_s

- α_s small at high q^2:

  High q^2 processes can be described perturbatively

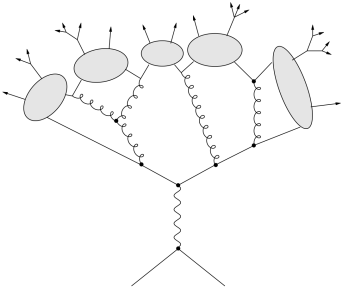
  - For DIS and $e^+e^- \rightarrow hadrons$, the lowest order process is electromagnetic or weak
  - Higher order perturbative QCD corrections can be added to the basic process
  - Processes we will discuss later (such as pp collisions), the lowest order process will be QCD
  - Again, can include perturbative corrections

- α_s large at low q^2:

  Quarks dress themselves as hadrons with probability=1 and on a time scale long compared to the hard scattering

  - Describe dressing of final quark and antiquark (and gluons if we consider higher order corrections) into a "Fragmentation Function"
  - Process of quarks and gluons turning into hadrons is called *hadronization*

Hadronization as a Showering Process



- Similar description to the EM shower that you modeled in HW# 1
  - Quarks radiate gluons
  - Gluons make $q\overline{q}$ pairs, and can also radiate gluons
- Must in the end produce color singlets
  - Nearby q and $\overline{q}$ combine to form clusters or hadrons
  - Clusters or hadrons then can decay
- Warning: Picture does not make topology of the production clear
  - Gluon radiation peaked in direction of initial partons
  - Expect collimated "jets" of particles following initial partons

Discovery of Jet Structure: Strategy

- While jets are clearly visible by eye at high energy, not the case for original experiments at low $\sqrt{s}$
- Discovery of jet structure required a statistical analysis using a global metric
  - Is the event spherical (as phase space would predict) or does it have a defined axis (the directions of the initial quark and anti-quark)?
- Define Sphericity Tensor

$$M_{ab} = \sum_{i}^{N} p_{ia} p_{ib}$$

  where a and b label x, y and z axes and the sum over i is a sum over all the (charged) particles in the event
- This looks just like a moment of inertia tensor
  - The relative value of the 3 eigenvalues tell us about the shape

Eigenvalues of the Sphericity Tensor

- From previous page: Sphericity Tensor

$$M_{ab} = \sum_{i}^{N} p_{ia} p_{ib}$$

- Define the 3 normalized eigenvalues:

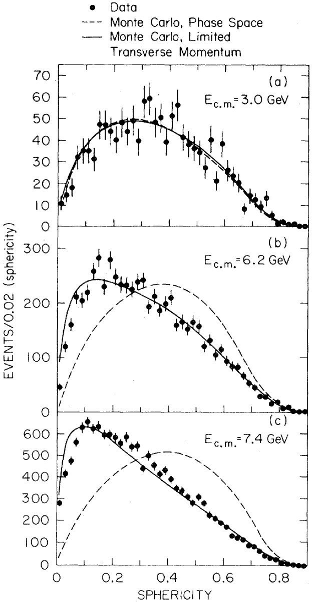
$$Q_k \equiv \frac{\Lambda_k}{\sum_i^N p_i^2}$$

where Λ_k are the 3 eigenvalues of the matrix

- The principle axis $\hat{n}_3$ is defined to be the jet axis
  - Method designed to identify narrow back-to-back jets
- Define the sphericity S

$$S = \frac{3}{2}(Q_1 + Q_2) = \frac{3}{2} \sum_i \frac{(p_{T,i}^2)_{min}}{\sum_i p_i^2}$$

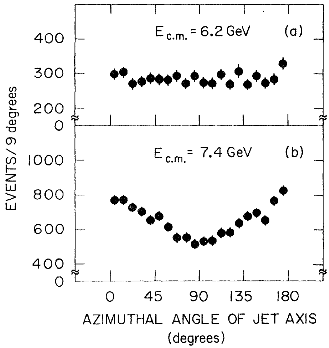
Emergence of Jets



Phys. Rev. Lett. **35**, 1609 (1975)

- Data collected by Mark-I experiment at SPEAR e^+e^- collider
- Study sphericity distribution for different E_{cm}
- Compare to a jet model and a phase space model
- As E_{cm} increase, data becomes consistent with jet model
  - Not consistent with phase space

Angular dependence of jet axis (same paper)



- Assume jet axis provides estimate of direction of outgoing quarks
- Since quarks have spin-$\frac{1}{2}$, distribution in polar angle

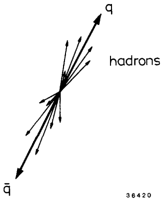
$$\frac{dN}{d\cos\theta} = 1 + \cos^2\theta$$

- But Mark-I had limited $\cos\theta$ coverage!
- But, if incoming beams transversely polarized, there is also a ϕ dependence

$$\frac{dN}{d\cos\theta} = 1 + \cos^2\theta + P_+ P_- \sin^2\phi\cos 2\phi$$

- Turns out that beams at SLAC were transversely polarized with polarization dependent on E_{cm}
- Angular dependence consistent with expectations for spin-1/2 Dirac particles

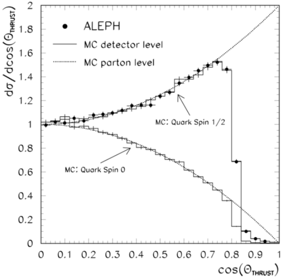
An alternative event shape variable: Thrust

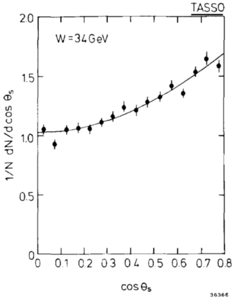


- Sphericity quadratic in p
  - Sensitive to hadronization details
- Linear alternative: Thrust axis

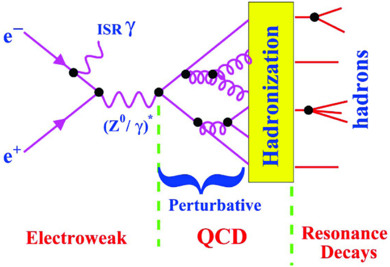
$$T = \max \frac{\sum |\vec{p}_i| \cdot \hat{n_T}}{\sum |\vec{p}_i|}$$

- Both choices appear to track quark direction well
  - Again, clear evidence for spin-$\frac{1}{2}$ quarks





QCD at Many scales



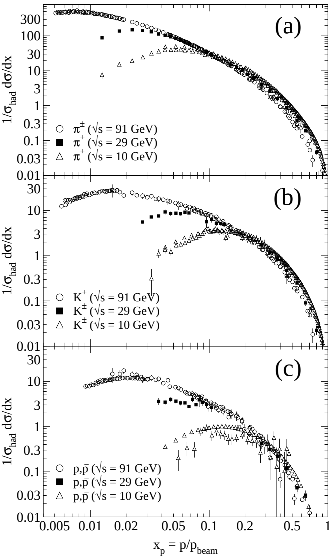
- Impulse approximation
  - Short time scale hard scattering (EM interaction in this case)
  - Perturbative QCD corrections (will discuss next time)
  - Long time scale hadronization process
- Approach to the hadronization:
  - Describe distributions individual hadrons statistically
  - Collect hadrons together to approximate the properties of the quarks and gluons they came from

Describe non-perturbative effects using a phenomonological model

Hadronization and Fragmentation Functions

- Define distribution of hadrons using a "fragmentation function":
  - Suppose we want to describe $e^+e^- \to h\,X$ where h is a specific particle (eg π^-)
  - Need probability that a q or $\overline{q}$ will fragment into h
  - Define $D_q^h(z)$ as probability that a quark q will fragment to form a hadron that carries fraction $z = E_h/E_q$ of the initial quark energy
  - We cannot predict $D_q^h(z)$
    - Measure them in one process and then ask are they universal
- These $D_q^h(z)$ are essential for Monte Carlo programs used to predict the hadron level output of a given experiment ("engineering numbers")
- But in the end, what we really care about is how to combine the hadrons to learn about the quarks and gluons they came from

Fragmentation Functions Measured in e^+e^- Annihilation



- Once momentum of hadron well above its mass, $D_q^h(z)$ almost independent of $\sqrt{s}$
  - Fragmentation functions exhibit scaling with logrithmic dependence on $\sqrt{s}$
- Overall charged multiplicity

$$< N_h >= \int_{z_{min}}^{1} F(z)dz$$

- A common parameterization of $F(z)$:

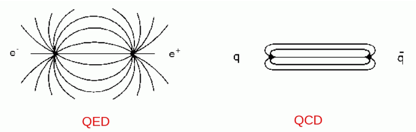
$$F(z) = N\frac{(1-z)^n}{z}$$

where n is a fitted parameter

- For this parameterization

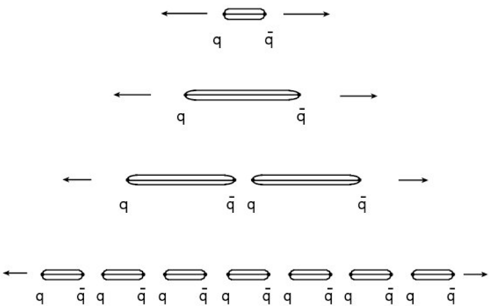
$$< N >= (n+1) < z >$$

Another Way of Thinking About Hadronization



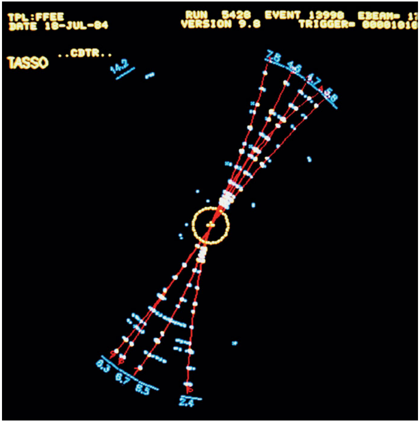
- q and $\overline{q}$ move in opposite directions, creating a color dipole field
- Color Dipole looks different from familiar electric dipole:
  - Confinement: At low q^2 quarks become confined to hadrons
  - Scale for this confinement, hadronic mass scale: $\Lambda =$ few 100 MeV
  - Coherent effects from multiple gluon emission shield color field far from the colored q and $\overline{q}$
  - Instead of extending through all space, color dipole field is flux tube with limited transverse extent
- Gauss's law in one dimensional field: E independent of x and thus $V(x_1 - x_2) = k(x_1 - x_2)$ where k is a property of the QCD field (often called the "string tension")
  - Experimentally, $k = 1$ GeV/fm $=$ 0.2 GeV^{-2}
  - As the q and $\overline{q}$ separate, the energy in the color field becomes large enough that $q\overline{q}$ pair production can occur
  - This process continues multiple times
  - Neighboring $q\overline{q}$ pairs combine to form hadrons

Color Flux Tubes



- Particle production is a stocastic process: the pair production can occur anywhere along the color field
- Quantum numbers are conserved locally in the pair production
- Appearence of the q and $\overline{q}$ is a quantum tunneling phenomenon: $q\overline{q}$ separate eating the color field and appear as physical particles

Jet Production



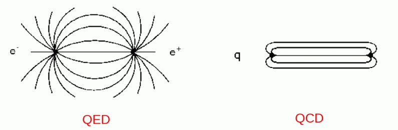
- Probability for producing pair depends quark masses

$$\text{Prob} \propto e^{-m^2/k}$$

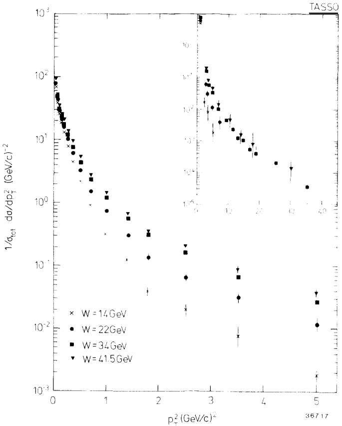
  relative rates of popping different flavors from the field are
  $u : d : s : c = 1 : 1 : 0.37 : 10^{-10}$

- Limited momentum tranverve to $q\overline{q}$ axis
  - If q and $\overline{q}$ each have tranverse momentum $\sim \Lambda$ (think of this as the sigma) the mesons will have $\sim \sqrt{2}\Lambda$
  - Meson transverse momentum (at lowest order) independent of qq center of mass energy
  - As E_{cm} increases, the hadrons collimate: "jets"

Characterizing hadronization using e^+e^- data: Limited Transverse Momentum



- q and $\overline{q}$ move in opposite directions, creating a color dipole field
  - Confinement limits transverse dimensions of the field
- Limited p_T wrt jet axis
  - $\sqrt{< p_T^2 >} \sim 350$ MeV
  - Well described by Gaussian distribution
- Range of longitudinal momenta (see next page)



SO [4.1] normalized differential cross section for the square of the momentum component transverse to the jet axis (= sphericity / = 14, 22, 34 and 41.5 GeV.

Characterizing hadronization using e^+e^- data: Rapidity and Longitudinal Momentum

- Define new variable: rapidity

$$y = \frac{1}{2}\ln\frac{E + p_{||}}{E - p_{||}}$$

Warning: Not the same y as in DIS

- Phase space with limited transverse momentum:

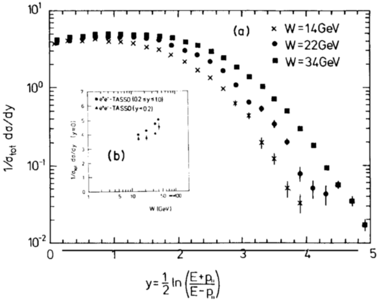
$$\frac{d^3p}{E} \rightarrow e^{-p_T^2/s\sigma^2} dp_T \frac{dp_{||}}{E}$$

- But

$$dy = \frac{dp_{||}}{E}$$

(you will prove this on HW #6)

- Rapidity is a longitudinal phase space variable



- Particle production flat in rapidity
- y_{max} set by kinematic limit $(E - p_{||}) \geq m_h$
- Height of plateau independent of $\sqrt{s}$
  - Multiplicity increase due to change in y_{max}
  - $< N_h > \sim \ln(\frac{E_{cm}}{m_h})$

Hadronization: Particle Multiplicity

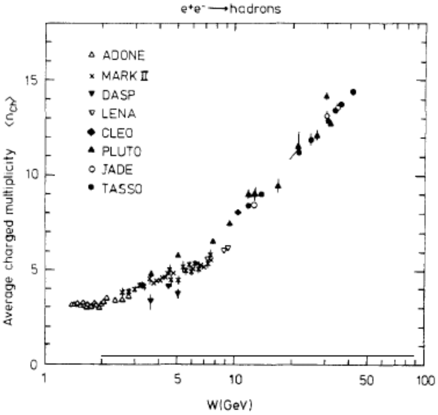
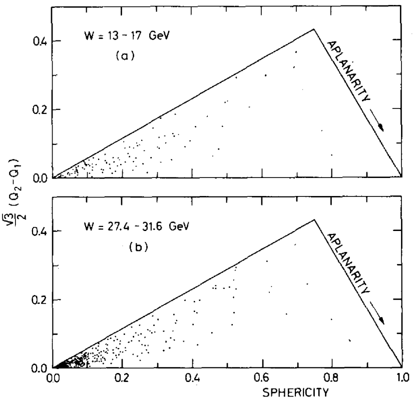


Fig. 4.1. Energy dependence of the average charged multiplicity.

- HW #6 will include derivation of $< N_h > \sim \ln(\frac{E_{cm}}{m_h})$
- This expression holds for E_{cm} above a few GeV

More on the sphericity tensor



- At SPEAR, seeing jets was difficult
  - Fixed transverse spread and small longitudinal momentum means the jets are wide
- As the energy increases, jets narrow: can look for wide angle gluon emission (3-jet events)
- QCD brem cross section diverges for colinear gluons or when the gluon momentum goes to zero
  - But that is the case where we can't distinguish 2 and 3 jet events anyway
  - Total cross section is finite (QCD corrections to R)
- Can use the sphericity tensor to search for 3-jet events (gluon brem)